Carl Friedrich Gauss

Michele Nichelatti

Oncology and Haematology Department, Niguarda Ca' Granda Hospital, Milan, Italy

Johann Carl Friedrich Gauss was born on 30 April 1777 in Braunschweig, part of the Duchy of Lüneburg, and now a city in the Land Niedersachsen, in Germany. Carl was the only son of a very poor and uneducated family: his father, Gerhard, like his grandfather, was a gardener, whereas his mother, Dorothea Benz, who died of tuberculosis at the age of 30, was the daughter of a stone-mason.

Carl's great gift became evident when he was just five years old. According to a famous anecdote (1), his primary school teacher, Herr Buttner, tried to occupy his pupils by asking them the sum of the first hundred integers. After few seconds, Carl stood up claiming he had got the right answer: fünftausend-und-fünfzig (5050).

It is not fully clear how he reached the right solution, but the simplest way, would have been first to write the integers in a row, then to write the same integers again, in reverse order, in a second row, so as to have

1	2	3	…	98	99	100
100	99	98	…	3	2	1

and, finally to sum the numbers in each column, to get

1	2	3	…	98	99	100
100	99	98	…	3	2	1
101	101	101	…	101	101	101

Thus, considering twice the sum of the integers paired in the reverse order, he arrived at $101 \times 100 = 10100$, hence to obtain the solution, he had to take this number and divide it by two, to get $10100 / 2 = 5050$. In other words, Carl rediscovered the finite arithmetic series

$$1 + 2 + 3 + ... + (n - 2) + (n - 1) + n = \frac{n\,(n + 1)}{2},$$

first used by the Persian algebraist Ibn al-Haytham, nearly eleven centuries before Gauss was born: not bad at all for a five-year-old boy, although some authors do not think this anecdote can be true (2).

In 1788, with the help of Büttner, his old teacher, Carl began his education at Gymnasium; then, in 1792, he was awarded a fellowship by the Duke of Braunschweig-Wolfenbüttel and entered the Collegium Carolinum (now renamed the Technische Universität Braunschweig), which he attended until 1795. He then studied at Göttingen University from 1795 to 1798.

During his studies, Gauss, by himself, discovered some important mathematical relationships, like the binomial theorem; he introduced the modular arithmetic (often called "the arithmetic of clock calculators") and made the fundamental discovery relating to triangular numbers, showing that any integers can be obtained by summing at most three triangular numbers (the comment he wrote in his diary "EYPHKA! num = $\Delta + \Delta + \Delta$" became famous). In 1798, at the age of 21, he wrote one of the most important books ever written in the field of mathematics, entitled *Disquisitiones Arithmeticae*, which was not published until 1801, due to financial problems affecting the publisher.

Gauss presented his dissertation, *Demonstratio nova theorematis omnem functionem algebraicam rationalem integram unius variabilis in factores reales primi vel secundi gradus revolvi posse*, in 1799 at Helmstedt University. In this work, Carl presented the first proof (as opposed to new proof to be added to pre-existing proof, as is clear from the title) of the fundamental theorem of algebra, showing that every polynomial in the field **C** of complex numbers possesses at least one root. Over his lifetime, Gauss produced three more different demonstrations of the same theorem, of which the last, written in 1849, is generally considered the most rigorous, according to modern standards.

On first January 1801, the Italian astronomer Giuseppe Piazzi discovered the new planetoid Ceres; Piazzi was unlucky, since he was able to track only three degrees of its orbit (just a couple of months) before it disappeared behind the Sun. Some mathematicians, including Gauss, were challenged by the problem of finding the position of Ceres: the various predictions were collected in June 1801, in a paper written by Franz Xaver von Zach, an astronomer whom Gauss had met a few years previously. It was von Zach who rediscovered Ceres on 31 December 1801 in Gotha, in almost the same position (the error was about equal to 1 second degree) calculated by Gauss. One day later, Ceres was found also by Heinrich Olbers (the one of the "starry night" paradox) in the Bremen observatory, confirming the exact prediction made by Gauss. It was the first success of the least squares method, even though Gauss did not explicitly present this among the calculations he made. He went on to prove this method rigorously in 1809, assuming the normal distribution of errors; in the meanwhile, French mathematician Adrien-Marie Legendre, in 1805, was the first to publish a description of this method, which Gauss confirmed he had been using since 1795.

The basic problem of least squares is to estimate the unknown parameter $p \times 1$ vector $\boldsymbol{\theta}$ from the equation $\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$, $\mathbf{y}$ being a $n \times 1$ vector of observations, $\mathbf{X}$ a $n \times p$ matrix, and $\boldsymbol{\varepsilon}$ the $n \times 1$ error vector. By means of the minimisation of $\sum_{k=1}^{n} \boldsymbol{\varepsilon}^2 = (\mathbf{y} - \mathbf{X}\boldsymbol{\theta})^T (\mathbf{y} - \mathbf{X}\boldsymbol{\theta})$, one obtains the least squares equation $(\mathbf{X}^T\mathbf{X})\hat{\boldsymbol{\theta}} = \mathbf{X}^T\mathbf{y}$ and the estimation $\hat{\boldsymbol{\theta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$. Legendre derived the general form of least squares equation (indeed, in a more generalised model, the matrix $\mathbf{X}$ must be replaced by a given known function $f(\boldsymbol{\theta})$ of the parameter vector), as well as the estimate $\hat{\boldsymbol{\theta}}$, and also coined the term "méthode des moindres quarrés" to define this technique. Gauss assumed the errors as independent random variables, with density function proportional to the likelihood function of $\boldsymbol{\theta}$, obtaining $\hat{\boldsymbol{\theta}}$ as a maximum likelihood estimate (3).

Gauss developed the final form of the concept of normal (now Gaussian) distribution, obtaining the curve

$$f(\mathrm{x}) = \frac{1}{\sigma\sqrt{2\pi}} exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

while working on a geodesic survey of the state of Hannover: this work was also important in developing his interest in differential geometry, and in the evolution of its famous *theorema egregium*.

Gauss was also very interested in linear algebra. The Gaussian elimination is a well-known method for solving linear systems. Gauss's scientific interests covered most other fields of pure and applied mathematics: his collaboration with Wilhelm Weber in the field of electromagnetism was in particular very fruitful.

In all his activities, Gauss showed a strong attitude for perfectionism and his work was also a way to forget his unhappy family life. He married twice and had three sons from each wife. The first one, Johanna Osthoff, died in 1809; a few months later Luis, another of his children, died, plunging Gauss into a deep depression. His second wife Friederica Wilhelmine Waldeck, died in 1831 after a long illness. Of Gauss's six children, none, in spite of their father's opposition, followed their father's career: in particular, two of them left Europe to go live in St. Louis (MO, USA), where one became a tycoon in the shoe manufacturing business.

Gauss was in general a gentle person, and had good relationships with other people. Nevertheless, some of his attitudes were, to say the least, unusual: Asimov (4) reports a famous anecdote regarding Gauss, who, when interrupted in the middle of a problem to be told by a servant that his wife was dying, simply answered: "tell her to wait a moment until I'm through".

References

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